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Color Group Approach to Symmetrical Ferrite Devices with Polarization Effects

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Abstract—**Applicability of the color group approach to symmetrical ferrite devices with polarization effects is demonstrated. By using the theory of symmetry, the scattering matrices of square and circular waveguides with quadrupole dc magnetic field are derived.**

The color (magnetic) group approach proposed in [1] is also applicable to symmetrical ferrite devices with polarization effects. To demonstrate this possibility, let us consider a square quadrupole gyromagnetic waveguide depicted in Fig. 1.

We shall use here a description of the waveguide in terms of a pair of orthogonal coupled modes [2] with the port nomenclature, illustrated in Fig. 1(b).

The square waveguide is described by the group of symmetry D_{4h} in Schoenflies notation [3]. The quadrupole dc magnetic field has the symmetry $D_{4h}(D_{2d})$. Hence, the magnetic group of symmetry of the system "waveguide + dc magnetic field" is $D_{4h}(D_{2d})$, and the group consists of 16 elements:

E	Identity operation.
S_{4z}, S_{4z}^{-1}	Improper rotation about z-axis by $\pi/2$ and $-\pi/2$, respectively.
C_{2z}	Rotation about z-axis by π.
$\sigma(a\text{-}a), \sigma(b\text{-}b)$	Reflection in the planes passing through the waveguide axis and the axes $(a\text{-}a)$ and $(b\text{-}b)$, respectively.
C_{2x}, C_{2y}	Rotation by π about x- and y-axis, respectively.
TC_{4z}, TC_{4z}^{-1}	Antirotation about z-axis by $\pi/2$ and $-\pi/2$, respectively.
$TC_{2(a-a)}, TC_{2(b-b)}$	Antirotation by π about $a\text{-}a$ and $b\text{-}b$ axes, respectively.
$T\sigma_x, T\sigma_y$	Antireflection in the planes $x = 0$ and $y = 0$, respectively.
$T\sigma_z$	Antireflection in the plane $z = 0$.
Ti	Anti-inversion.

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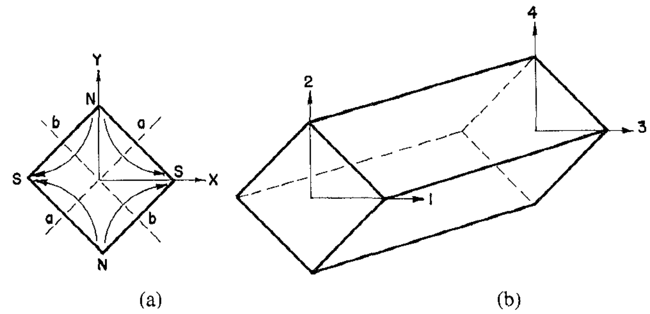

Fig. 1. Schematic diagram of square waveguide with a quadrupole dc magnetic field.

Here T denotes the operation of time reversal.

Generators of the group may be, for example, $C_{2x}, \sigma_{(a-a)}$ and $T\sigma_z$. The corresponding symmetry operators of the quadrupole gyromagnetic waveguide are

$$[R]_{C_{2x}} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}; \quad [R]_{\sigma_{(a-a)}} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix};$$

$$[R]_{T\sigma_z} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}.$$

Using the following commutation relations

$$[R]_{C_{2x}}[S] = [S][R]_{C_{2x}},$$
$$[R]_{\sigma_{(a-a)}}[S] = [S][R]_{\sigma_{(a-a)}},$$
$$[R]_{T\sigma_Z}[S] = [S]^t[R]_{T\sigma_z},$$

where t denotes transposition, we get the matrix $[S]$

$$[S] = \begin{vmatrix} S_{11} & 0 & S_{13} & S_{14} \\ 0 & S_{11} & S_{14} & S_{13} \\ S_{13} & -S_{14} & S_{11} & 0 \\ -S_{14} & S_{13} & 0 & S_{11} \end{vmatrix}. \tag{1}$$

The matrix $[S]$ has three independent complex parameters. The quantities S_{ij} depend on the parameters of the waveguide and its length. Notice that in the matrix (1) $S_{13} = S_{31}, S_{24} = S_{42}$, because the pairs of the ports 1, 3 and 2, 4 lie in the antiplanes of symmetry $y = 0$ and $x = 0$, respectively. It is in accordance with the theory suggested in [1]. The ports 1 and 2, and also 3 and 4 are completely decoupled from each other.

If $S_{11} = 0$, we get the scattering matrix for the ideally matched waveguide. Then, using the unitary conditions

$$[S][S^*]^t = [E] = [S^*]^t[S]$$

we get the matrix $[S]$ for the waveguide without losses

$$[S] = \begin{vmatrix} 0 & 0 & k & \pm j\sqrt{1-k^2} \\ 0 & 0 & \pm j\sqrt{1-k^2} & k \\ k & \mp j\sqrt{1-k^2} & 0 & 0 \\ \mp j\sqrt{1-k^2} & k & 0 & 0 \end{vmatrix}. \tag{2}$$

This matrix describes a nonreciprocal directional coupler. Here, with properly chosen terminal surfaces, $k = S_{13}$ is real. The upper

$$[S] = \begin{vmatrix} 0 & 0 & S'_{13} - S'_{14}\sin 2\theta & S'_{14}\cos 2\theta \\ 0 & 0 & S'_{14}\cos 2\theta & S'_{13} + S'_{14}\sin 2\theta \\ S'_{13} + S'_{14}\sin 2\theta & -S'_{14}\cos 2\theta & 0 & 0 \\ -S'_{14}\cos 2\theta & S'_{13} - S'_{14}\sin 2\theta & 0 & 0 \end{vmatrix}$$

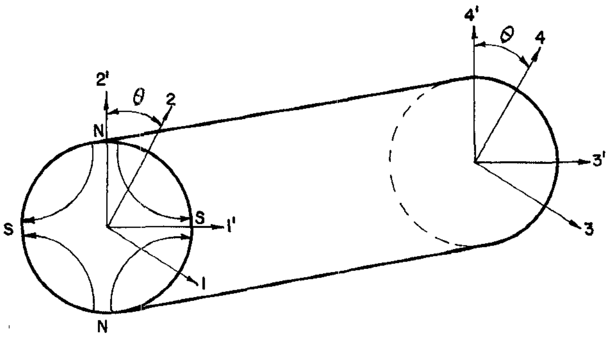

Fig. 2. Schematic diagram of circular waveguide with a quadrupole dc magnetic field.

and lower signs in the matrix (2) correspond to two directions of dc magnetic field or two directions of propagation. When $k = \sqrt{2}/2$, we have the case of a quarter wave plate.

Let us apply now to a circular gyromagnetic waveguide (Fig. 2). Write down the matrix $[S']$ for the ideally matched waveguide with the ports $1', 2', 3'$, and $4'$, lying in the antiplanes of symmetry

$$[S'] = \begin{vmatrix} 0 & 0 & S'_{13} & S'_{14} \\ 0 & 0 & S'_{14} & S'_{13} \\ S'_{13} & -S'_{14} & 0 & 0 \\ -S'_{14} & S'_{13} & 0 & 0 \end{vmatrix}.$$

To find the matrix $[S]$ for the waveguide with the ports 1, 2, 3, and 4, rotated at an angle θ about the ports $1', 2', 3'$, and $4'$, consider, for example, a wave a_1 in the port 1. It may be presented as a vector sum of the two components a'_1 and a'_2 in the ports $1'$ and $2'$

$$a'_1 = a_1 \cos\theta, \quad a'_2 = -a_1 \sin\theta.$$

Using the matrix relation between reflected and incident waves

$$[b'] = [S'][a']$$

we have

$$b'_3 = a_1 S'_{13}\cos\theta + a_1 S'_{14}\sin\theta$$
$$b'_4 = -a_1 S'_{13}\sin\theta - a_1 S'_{14}\cos\theta.$$

The sum of the projections of b'_3 and b'_4 on the port 3 is

$$b_3 = b'_3\cos\theta - b'_4\sin\theta.$$

Now we may derive the element S_{31}

$$S_{31} = b_3/a_1 = S'_{13} + S'_{14}\sin 2\theta.$$

The other elements of $[S]$ are calculated analogously. Thus, the desired scattering matrix has the form as in the matrix shown at the top of the page.

The elements S_{13}, S_{31}, S_{24}, and S_{42} consist of two parts: reciprocal S'_{13} and nonreciprocal $S'_{14}\sin 2\theta$. The latter depends on the angle θ.

When $S'_{13} = 0$, we obtain the matrix for the quadrupole half-wave plate

$$[S] = S'_{14}\begin{vmatrix} 0 & 0 & -\sin 2\theta & \cos 2\theta \\ 0 & 0 & \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta & 0 & 0 \\ -\cos 2\theta & -\sin 2\theta & 0 & 0 \end{vmatrix}. \quad (3)$$

If $|S'_{14}| = 1$, it is the case of a nondissipative waveguide. With $\theta = \theta' + \pi/4$ and $S'_{14} = 1$ the matrix (3) is transformed into the matrix, deduced in [2] by another method.

The method is applicable to symmetrical devices with gyroelectric media as well.

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A New Approach for Analysis of Resonant Structures Based on the Spatial Finite-Difference and Temporal Differential Formulation

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***Abstract*—This paper presents a new procedure for analyzing resonant structures using the spatial finite-difference and temporal differential formulation. Unlike the conventional finite-difference time-domain methods, the finite-difference are only enforced in the spatial domain for Maxwell's equations. The time-domain differentials of Maxwell's equations are kept, resulting in a system of first-order differential equations. In consequence, a resonant structure problem can be formulated in the eigenvalue problem form and resonant modes are obtained by solving the corresponding eigenvalue problem directly. It is shown that the coefficients of the matrix for the eigenvalue problem can be simply obtained from the finite-difference time-domain formulation. As a result, an efficient alternative way of using the finite-difference time-domain approach to solve the resonant structure problems is presented. The algorithm is applied to metallic waveguide structures and the numerical results agree well with those from other techniques.**

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